Harmonization of Interests for Sustainable Developement *

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Abstract **Multilevel decision-making systems, are considered in a framework of sustainable developement. The property of existence of an optimal (in some special senses) vector of all system's controls is called the harmonizability of interests of the parties. A few possible statements of the harmonization problems are presented and some conditions of their solvability are given.**

I. INTRODUCTION

The problem of harmonization of interests is actual in all cases when there are at least two parties, each of the parties pursuing its goal. In this case, harmonization is necessary to eliminate conflicts via achieving the compromise. Examination of harmonizability or finding out the conditions, under which harmonization of interests is possible, is a very essential problem of sustainable developement. At the same time, the intuitive concept of harmonization itself, which is informal, presumes a number of different aspects, therefore there exists a lot of mathematical definitions of this concept, each of them reflecting some aspect of the harmonization problem.

Consider some of known approaches to the harmonization problem. For a hierarchical two-level system, which is composed of a coordinating center and a few 2d level subsystems, M.Mesarovic proposed [1] a concept of coordination. Such a system is called coordinable if there exists a control of the center to the subsystems, under which the local control of each of the subsystems maximizes its own quality criterion, and the set of these local controls simultaneously maximizes the global quality criterion for the center.

Some elements of intuitive understanding of the harmonization concept are contained also in the definition of Nash equilibrium (in a noncooperative game of a few players). In cooperative games, harmonization of interests may be realized by the maximum extension of coalitions and via dynamically stable sharing the gain (L. Petrosyan [2]). In mathematical programming, a problem of coordination of regional and branch plans which is rather similar to the problem of Mesarovic coordinability, has been considered (K. Bagrinovsky [3]),. A valuable contribution to the problems under consideration has been introduced also by Hermeier-Moiseev's school, this particularly, concerns the finding of Nash strong stability conditions, furthermore, both in static and dynamic statements (Yu.Hermeier, I.Vatel [4], A.Kononenko [5], E.Konurbayev [6]).

This paper proposes a few new definitions of harmonization, which are of interest for practice and formalize some other aspects of the harmonization problem for dynamic systems. First results of inverstigation of corresponding problems are given.

II. OPTIMIZATION APPROACH

This section presents some statements for the problems of the type of "Local administration-Industrial enterprise".

Consider a system whose behavior is described by an ordinary differential (or difference) equation:

$$\dot{x} = \Phi(t, x, u), \quad x \in R^n, t \in T = [t_0, t_1] \qquad (1)$$

$$(\text{or } x(t+1) = \Phi(t, x(t), u), \ t = 1, 2, ...),$$

where u is the control, $u = (v, w) \in V \times W \subseteq R^r \times R^s$, i.e. there are two players, each of them governs its finite-dimensional control : the first governs the control v, the second - w. Suppose, the first player may choose its control v depending on the second player's control w (laws, norms and rules of taxation, penalties, investment, etc. allow variations for the purposes of more flexible influence on the business activity, purity (pollutionlessness) of technologies, etc.), i.e. the control v will be considered to be represented by the function $v : T \times R^n \times W \to V$, which belongs to a class of admissible functions $\mathcal{V}$ (for example, continuous, piecewise-continuous and other functions).

The second player's control will be considered as depending only on time, i.e. $w : T \to W$, from some class $\mathcal{W}$ (for example, continuous, measurable, etc.).

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Henceforth, we also assume that the initial moment t_0 and the initial state x_0 are fixed, and the right-hand side Φ of eq. (1) and the classes of functions $\mathcal{V}$ and $\mathcal{W}$ are chosen such that for any $v \in \mathcal{V}$, $w \in \mathcal{W}$ there exists a unique and continuable on T K-solution (solution in the sense of Carateodory) of the initial value problem

$$\begin{cases} \dot{x} = \Phi(t, x, v(t, x, w(t)), w(t)), \\ x(t_0) = x_0 \end{cases} \tag{2}$$

which will be denoted by $x(\cdot, t_0, x_0, v, w)$ (or by $x(t, v, w)$). Suppose also that interests of each of the players are represented by the corresponding vector quality criteria

$$\begin{array}{l} \mathcal{J} : \mathcal{V} \times \mathcal{W} \to R^{m_1} - \text{for the 1st player,} \\ \mathcal{I} : \mathcal{V} \times \mathcal{W} \to R^{m_2} - \text{for the 2d player.} \end{array} \tag{3}$$

For each fixed $\tilde{v} \in \mathcal{V}$ we will also consider the criteria $\mathcal{J}(\tilde{v}, \cdot)$ and $\mathcal{I}(\tilde{v}, \cdot)$ as the functionals defined on $\mathcal{W}$, and denote the Pareto sets of the first and the second players with respect to these functionals by, respectively, $\mathcal{P}_{\mathcal{J}}(\tilde{v})$ and $\mathcal{P}_{\mathcal{I}}(\tilde{v})$. Let us also denote by $\mathcal{P}_{\mathcal{J}}$ the Pareto set for the functional $\mathcal{J}$ as a subset from $\mathcal{V} \times \mathcal{W}$.

Definition 1 (strong harmonization). Let us speak that the property of strong harmonization (SH) holds for the problem (1), (3) iff

$$(\exists v \in \mathcal{V})(\mathcal{P}_{\mathcal{I}}(v) \neq \emptyset \ \& \ \forall w \in \mathcal{W}(w \in \mathcal{P}_{\mathcal{I}}(v) \to$$

$$\to (v, w) \in \mathcal{P}_{\mathcal{J}})).$$

Note, in the scalar case ($m_1 = m_2 = 1$), the property (SH) may be written in the form

$$(\exists v \in \mathcal{V})[(\exists w_0 \in \mathcal{W} \ \ \mathcal{I}(v, w_0) = \max\{\mathcal{I}(v, w') \mid$$

$$w' \in \mathcal{W}\})\&(\forall w \in \mathcal{W} \ \ \mathcal{I}(v, w) = \max\{\mathcal{I}(v, w') \mid$$

$$w' \in \mathcal{W}\} \Rightarrow \mathcal{J}(v, w) = \max\{\mathcal{J}(v', w') \mid v' \in \mathcal{V}\&w' \in \mathcal{W}\}]$$

Let us propose the existence criterion for the property (SH) for this case by using a combination of a Lyapunov-Bellmann-type function and some auxiliary equation (comparison system).

Theorem 1. Let $m_1 = m_2 = 1$ and the functional $\mathcal{J}$ be terminal, i.e. $\mathcal{J}(v, w) = F(x(t_1, v, w))$, where $F : R^n \to R^1$.

Let also for some $v_0 \in \mathcal{V}$ there exists a continuous differentiable function $\varphi : T \times R^n \to R^1$ and a scalar system (comparison system)

$$\begin{cases} \dot{y} = \Phi_c(t, y, w), \\ y(t_0) = \varphi(t_0, x_0) \end{cases} \tag{4}$$

where $\Phi_c : T \times R^1 \times W \to R^1$, and the class of controls $w \in \mathcal{W}$ is the same as for the second player, and with the quality criterion $\mathcal{I}_c(w) = y(t_1, w)$, which are such that the following conditions hold.

1° . There exists at least one solution of the optimal control problem $\max\{\mathcal{I}(v_0, w) \mid w \in \mathcal{W}\}$ for the system

$$\begin{cases} \dot{x} = \Phi(t, x, v_0(t, x, w), w), \\ x(t_0) = x_0. \end{cases} \tag{5}$$

2° . Let for any $w \in \mathcal{W}$ there exists a unique and continuable on T solution of the initial value problem (4) and Carateodory conditions for the function $\Phi_c(t, y, w(t))$ hold.

3° . If some $w_0 \in \mathcal{W}$ is optimal in the problem (5), then it is optimal also for the comparison system (4), i.e. it maximizes $\mathcal{I}_c$.

4°. $(\partial\varphi(t, x)/\partial x, \ \Phi(t, x, v_0(t, x, w), w)) = \max_{v \in V}(\partial\varphi(t, x)/\partial x), \Phi(t, x, v, w)$.

5°. $\partial\varphi(t, x)/\partial t + (\partial\varphi(t, x)/\partial x, \Phi(t, x, v_0(t, x, w), w)) = \Phi_c(t, \varphi(t, x), w)$.

6°. $\varphi(t_1, x) = F(x)$.

Then the control v_0 provides for satisfaction of the strong harmonization property for the problem (1), (3).

Remark. Conditions 2°, 4°, 5° and 6° are ordinary for comparison method theorems [7] , furthermore, 4° and 5° provide the satisfaction of both differential inequalities on any controls and differential equalities on optimal ones, and from 6° it follows that $\mathcal{J}(v_0, w_1) = \mathcal{I}_c(w_1)$.

Condition 1° may be provided by standard conditions of existence of an optimal solution of the type of compactness, convexity, etc.

Condition 3° is the comparison property for the property (SH) and may be simplified, in some sense, for further investigations. For example, we may require optimality in (4) for all controls satisfying Pontryagin's maximum principle for the problem (5) (in the case when the criterion $\mathcal{I}$ is of Boltz type). This may turn out to be more easy verifiable (since the requirement of finding optimal solutions is eliminated in this case), or else we may replace the optimality condition for w_0 in the problem (5) with another condition necessary for optimality. On the other hand, if e.g. Φ_c does not depend on w (i.e., in Condition 5°, w does not enter in Φ_c), then Condition 3° is satisfied automatically.

Let us give definitions of other two properties, which are weaker than the property (SH) but also include some elements of the harmonization concept.

Definition 2 (Pareto maximization). Let us speak that the control v_0 solves the Pareto maximization (PM) problem iff $\mathcal{P}_{\mathcal{I}}(v_0) \neq \emptyset$ and for any $w_0 \in \mathcal{P}_{\mathcal{I}}(v_0)$, for any $(v, w) \in \mathcal{V} \times \mathcal{W}$ such that $w \in \mathcal{P}_{\mathcal{I}}(v)$

$$\mathcal{J}(v_0, w_0) \ \overline{\overline{>}} \ \mathcal{J}(v, w), \text{where } x \ \overline{\overline{>}} \ y \Leftrightarrow x = y \vee \exists i \ x_i > y_i.$$

In the scalar case ($m_1 = m_2 = 1$) , this property can be written as follows:

if $w_0 \in \text{argmax}\,\{\mathcal{I}(v_0, w) \mid w \in \mathcal{W}\} \neq \emptyset$ then $\mathcal{J}(v_0, w_0) = \max\,\{\mathcal{J}(v, w) \mid v \in \mathcal{V} \& w \in \text{argmax}\,\{\mathcal{I}(v, w') \mid w' \in \mathcal{W}\}\}$.

Definition 3 (weak harmonization). Let us speak that for the Problem (1), (3) the property of weak harmonization (WH) holds iff

$$(\exists v \in \mathcal{V})\ (\mathcal{P}_{\mathcal{I}}(v) \neq \emptyset\ \&\ \mathcal{P}_{\mathcal{I}}(v) \subseteq \mathcal{P}_{\mathcal{J}}(v)).$$

In the scalar case, the latter means that argmax $\{\mathcal{J}(v, w) \mid w \in \mathcal{W}\} \supseteq \text{argmax}\,\{\mathcal{I}(v, w) \mid w \in \mathcal{W}\} \neq \emptyset$.

Note, if the control $v_0 \in \mathcal{V}$ satisfies property (WH) then it will satisfy also the conditions of Definitions 2 and 3. At the same time, whereas property (PM), in the scalar case, may always be provided at least with the help of the minimizing sequence, properties (SH) and (WH) are not always satisfied.

III. OPTIMALITY WITH DYNAMICS

Above we considered an optimization approach to formalization of harmonizability of interests. On the other hand, it is possible to consider harmonizability as a complex dynamic controllability property which will be also called sustainable development (SD). Furthermore , such a property (typical of harmonizability) as elimination of sharp contrasts may be considered. Elimination of sharp contrasts must be: 1) "smooth", 2) secure, 3) purposeful, 4) stable.

The second and third aspects may be formalized as a dynamic property of controllability in some set G under phase constraints P at the moments of leaving G , furthermore, the time for which the trajectory leaves G shall not be large (shall be not more than Δ_1).

The first aspect ("smoothness") may be understood as almost monotonicity with respect to some quality criterion $f(u, x)$ along time, i.e. monotonicity may be violated but not more than by ϵ and not for long (the time interval of violated monotonicity is less than Δ_2). Here the control u belongs to a class $\mathcal{U}$ of admissible controls, and it is not necessery that $\mathcal{U}$ has a structure of production $\mathcal{V} \times \mathcal{W}$ (it is not necessery that $u = (v, w)$).

Let us give a precise formulation of the above property.

$$\begin{aligned}
&(\text{SD}) = \exists u \in \mathcal{U}\ \{\forall \mathrm{x}_0 \in \mathrm{X}^0\ \forall x(\cdot, t_0, \mathrm{x}_0, u)\ \exists t \geq t_0\\
&[x(t) \in G(t) \&\\
&\&\ \forall t_1 \geq t_0\ \ x(t_1) \in P(t_1) \&\\
&\&\ (\ \forall t_2 > t : x(t_2) \notin G(t_2)\ \ \forall t_3 > t_2 : x(t_3) \notin G(t_3)\\
&t_3 - t_2 < \Delta_1 \vee \exists t_4 \in [t_2, t_3]\ \ x(t_4) \in G(t_4)) \&\\
&\&\ (\forall t' \in [t_0, t]\ (\forall t" \in [t_0, t] : t' \leq t")\\
&f(t', x(t')) \leq f(t", x(t")) \vee\\
&\vee\ (t" - t' < \Delta_2\ \&\ f(t', x(t")) - f(t", x(t")) \leq \epsilon))]\}.
\end{aligned}$$

To obtain the criterion of existence of the property (SD) consider an auxiliary (simpler) system in R^k, which is called also the comparison system

$$\dot{\mathrm{x}}_c = \Phi_c(t, \mathrm{x}_c, u_c(t, \mathrm{x}_c)),\ \mathrm{x}_c \in R^k,\ u_c \in \mathcal{U}_c \qquad (6)$$

and the comparison property $(\text{SD})_c$, which is similar to the property (SD) [7]. The tuple (v, s) including of the function $\mathrm{v} : T \times R^n \rightarrow R^k$ and the map $s : \mathcal{U} \rightarrow \mathcal{U}_c$ will be called the vector comparison function (VCF) for systems (1), (6) if the connection condition of the following form holds

$$(\forall u \in \mathcal{U})\ (\forall \mathrm{x}_0 \in \mathrm{X}_0)\ (\forall t \in \text{dom} x \cap \text{dom} x_c)$$

$$\mathrm{v}(t, x(t, t_0, \mathrm{x}_0, u)) = x_c(t, t_0, \mathrm{x}_{0c}, u_c)$$

where $u_c = s(u), \mathrm{x}_{0c} = \mathrm{v}(t_0, \mathrm{x}_0)$.

Theorem 2 (comparison theorem for (SD)). Let for the systems (1), (6) there exists a VCF $(\mathrm{v}, s), \mathrm{v} : T \times R^n \rightarrow R^k,\ s : \mathcal{U} \rightarrow \mathcal{U}_c$ such that

1)$\mathrm{v}(\mathrm{X}_0) \subseteq \mathrm{X}_{0c},\ \mathrm{v}^{-1}(G_c) \subseteq G,\ \mathrm{v}^{-1}(P_c) \subseteq P$;

2) the solutions $x(\cdot, t_0, \mathrm{x}_0, u)$ of the system (1) are rihgt-continuable for all $\mathrm{x}_0 \in \mathrm{X}_0,\ u \in \mathcal{U}$; the solutions $x_c(\cdot, t_0, \mathrm{x}_{0c}, u_c)$ of the system (6) are right-continuable for all $\mathrm{x}_{0c} \in \mathrm{v}(\mathrm{X}_0),\ u_c \in s(\mathcal{U})$;

3) $\Delta_{1C} \leq \Delta_1,\ \Delta_{2C} \leq \Delta_2$;

4) $f_c(t', \mathrm{v}(t', \mathrm{x}')) \leq f_c(t", \mathrm{v}(t", \mathrm{x}")) \rightarrow f(t', \mathrm{x}') \leq f(t", \mathrm{x}")$;

5) $f_c(t', \mathrm{v}(t', \mathrm{x}')) - f_c(t", \mathrm{v}(t", \mathrm{x}")) \leq \epsilon_c \rightarrow f(t', \mathrm{x}') - f(t", \mathrm{x}") \leq \epsilon$.

Then $(\text{SD})_c \Rightarrow (SD)$.

IV.THE PROBLEM OF HARMONIZATION FOR A MODEL "LOCAL ADMINISTRATION - INDUSTRIAL ENTERPRIZE"

Consider the following model of functioning of an enterprize, for which the mechanism of penalties for perturbing ecological equilibrium is formalized.

Let time t vary discretely ($t = 0, 1, ...$), $v(t)$ be the annual mass output of products, $\Phi(t)$ be capital manufacturing funds. Suppose also that the relationship between v and Φ may be described by the Cobb-Douglas manufacturing function

$$0 \leq v(t) \leq k_{\Phi}^0 e^{\beta_\Phi t} (\Phi(t))^{\alpha_\Phi} = V(t) \qquad (7)$$

where $k_{\Phi}^0, \beta_\Phi, \alpha_\Phi$ are constants, $k_{\Phi}^0, \alpha_\Phi > 0, \mathrm{V}(t)$ is the production capacity of an enterprise (maximum possible output).

Denote by $\Phi^z(t)$ capital funds for purification devices of the enterprise, $z(t)$ - purification intensity, $Z(t)$ - capasity of purification devices. The latter functions are connected by the dependences

$$0 \le z(t) \le Z(t) = \kappa\Phi^z(t), \quad (8)$$

where $\kappa > 0$ is constant.

Suppose, the variation dynamics of funds is described by the equations

$$\Phi(t+1) = k_\Phi^1\Phi(t) + k^1 I(t) + k^2 I(t-1),$$
$$\Phi^z(t+1) = (1-\Delta)\Phi^z(t) + I^z(t), \quad (9)$$

where I and I^z are investments computed from the profit rate of the enterprise.

In order to determine the profit rate, let us introduce the following notation:

$L(t), L^z(t)$ - number of workers in the manufacturing and in the purification works, resp.;
$L(t) = \lambda v(t), L^z(t) = \lambda^z z(t),\ \lambda, \lambda^z$ - const.;
$M(t), M^z(t)$ - wages funds;
$M(t) = mL(t), M^z(t) = m^z L^z(t),\ m, m$ - const.;
$D(t), D^z(t)$ - depreciation rate;
$D(t) = d\Phi(t), D^z(t) = d^z\Phi^z(t),\ d, d^z$ - const.;
$av(t), a^z z(t)$ - direct manufacture and purification; a, a^z - const.;
$lL(t), l^z L^z(t)$ - cost of labour resources; l, l^z - const.;
$\varphi(t)$ - pollution;
$\varphi(t) = \xi v(t) - z(t),\ \ \xi$ - const.;
$S_r(\varphi)$ - penalty function.

Assume, the penalty function is

$$S_r(\varphi) = \begin{cases} c_1\varphi, & 0 \le \varphi \le k \\ \tilde c\varphi, & \varphi > k \\ 0, & \varphi < 0 \end{cases}$$

where $c_1, \tilde c, k$ - const, $\tilde c > c$, k is the threshold after which the penalties abruptly increase (other variants of the penalty function are possible).

The function $S_r(t)$ will be represented as the sum $S_r(\varphi) = S_r^1(\varphi) + S_r^2(\varphi)$, where

$$S_r^1(\varphi) = \begin{cases} c_1\varphi, & 0 \le \varphi \le k \\ c_1 k, & \varphi > k \\ 0, & \varphi < 0 \end{cases}, S_r^1(\varphi) = \begin{cases} 0, & \varphi \le k \\ \tilde c\varphi - c_1 k, & \varphi > k \end{cases}$$

Determine now the manufacturing cost of the production

$$S(t) = D(t) + D^z(t) + av(t) + a^z z(t) + M(t) + $$
$$+ M^z(t) + lL(t) + l^z L^z(t) + S_r^1(\varphi(t)) \quad (10)$$

and the total annual profit

$$\tilde\rho(t) = v(t) - S(t).$$

Let α be the payments to the regional budget , α is const, then the residual profit is

$$\rho(t) = (1-\alpha)\tilde\rho(t) - S_r^2(\varphi(t)).$$

Next, the residual profit is subdivided into the labor stimulation fund and the development fund:

$$\Phi_m(t) = \delta\rho(t), \Phi_r(t) = (1-\delta)\rho(t),\ \delta - \text{const}.$$

Then, total capital investments will write

$$I^0(t) = I^c(t) + \Phi_r(t),$$

where $I^c(t)$ are centralized capital investments; and after that are subdivided into production investments and purification investments:

$$I(t) = \gamma I^0(t),\ I^z(t) = (1-\gamma)I^0(t),\ \gamma - \text{const}.$$

Let us solve the problem of harmonization of interests between an enterprise and the local administration on each step from t to $t+1$. Furthermore, as the criteria, we choose the following:

- an enterprise tends to maximize its residual profit $\mathcal{I} = \rho(t+1)$;
- local administration tends to maximize some linear combination of production and pollution

$$\mathcal{J} = \eta_1 v - \eta_2\varphi,\ \eta_1, \eta_2 - \text{const}\ ,\eta_1, \eta_2 > 0,$$

i.e. to increase the mass output v and to reduce the pollution z.

On account of $\varphi = \xi v - z$, the criterion $\mathcal{J}$ may be written $\mathcal{J} = (\eta_1 - \eta_2\xi)v + \eta_2 z$, and it is reasonable to assume that $\eta_1 - \eta_2\xi > 0$, since, otherwise, the local administration will not be interested in increasing the production output.

Hence, we have the following problem.

Let the functions $\Phi(t), \Phi^z(t), I(t-1), \rho(t), I^c(t)$ be dermined on a step t. Then, for $I(t)$ and $I^z(t)$ we have

$$I(t) = \gamma[I^c(t) + (1-\delta)\rho(t)]$$

$$I^z(t) = (1-\gamma)[I^c(t) + (1-\delta)\rho(t)]$$

and from eqs. (9) it is possible to determine $\Phi(t+1)$, $\Phi^z(t+1)$ and also the production capacities $V_0 = V(t+1), Z_0 = Z(t+1)$. From eq. (10) we obtain the expression for $\rho(t+1)$:

$$\rho(t+1) = \rho(v(t+1), z(t+1)) =$$
$$= (1-\alpha)[\tilde\rho(t+1) - S_r^2(\varphi(t+1))/(1-\alpha)] =$$
$$= (1-\alpha)[v(t+1) - S(t+1) - S_r^2(\varphi(t+1))/(1-\alpha)] =$$
$$= (1-\alpha)[(1-\tilde a)v(t+1) - \tilde a^z z(t+1) -$$
$$- S_r^0(\xi v(t+1) - z(t+1)) - D^0(t+1)]$$

where $\tilde a = (m+l)\lambda + a,\ \tilde a^z = (m^z + l^z)\lambda^z + a^z$ the function S_r^0 has the structure:

$$S_r^0(\varphi) = \begin{cases} c_1\varphi, & 0 \le \varphi \le k \\ c_0(\varphi - k) + k(c_1 + c_2), & \varphi > k \\ 0, & \varphi < 0 \end{cases}$$

where $c_0 = \tilde{c}/(1-\alpha) > c_1$, $c_2 = (\tilde{c} - c_1)/(1-\alpha) > 0$, and $D^0(t+1) = D(t+1) + D^z(t+1) = d\Phi(t+1) + d^z\Phi^z(t+1)$ does not depend on $v(t+1)$, $z(t+1)$.

Consequently, for the enterprise under consideration we obtain the maximization problem for the function

$$\rho(v,z) = (1-\alpha)[(1-\tilde{a})v - \tilde{a}^z z - S_r^0(\xi v - z) - D^0]$$

under the constraints (7), (8), i.e.

$$0 \le v \le \mathrm{V}_0,\ 0 \le z \le \mathrm{Z}_0,$$

and for the local administration - the maximization problem for

$$\mathcal{J}(v,z) = (\eta_1 - \eta_2\xi)v + \eta_2 z$$

under the same constraints.

In this case, the function S_r is the control function of the local administration, i.e. the administration states the coefficients $c_1, \tilde{c}$, and the threshold k ; and the output v and the purification intensity z are controls assigned by the enterprise.

In accordance with Definition 1, the problem of strong harmonization in this case consists in finding such a function S_r that the set of tuples (v,z), in which maximum of $\rho(v,z)$ is achieved, be contained in the set of tuples (v,z), for which maximum of $\mathcal{J}(v,z)$ is achieved.

Theorem 3. The profitability condition $\tilde{a} + \tilde{a}^z\xi < 1$ is both the necessary and sufficient condition of solvability of the strong harmonization problem (SH) for a given model, furthermore, the function S_r solves this problem iff one of the following conditions holds:

1) $0 < \xi\mathrm{V}_0 - \mathrm{Z}_0 < k,\ \tilde{a}^z < c_1 < (1-\tilde{a})/\xi;$

2) $\xi\mathrm{V}_0 - \mathrm{Z}_0 = k,\ \tilde{a}^z \le c_1 < (1-\tilde{a})/\xi;$

3) $k[1 + \xi(\tilde{c} - c_1)/((1-\tilde{a})(1-\alpha) - \tilde{c}\xi)] <$

$< \xi\mathrm{V}_0 + \mathrm{Z}_0,$

$\tilde{a}^z < \tilde{c}/(1-\alpha) < (1-\tilde{a})/\xi.$

V. CONCLUSION

We have introduced a few new mathematical definitions of the intuitive concept of interests harmonization for the players. In future it is also interesting to include in formalization of the concept some aspects of time variability of the quality criterion itself.

We have also stated some conditions for existence of these properties, particularly, in terms of Lyapunov-like functions. It is important to develop some constructive procedures of finding these functions. For some classes of nonlinear systems and for their properties of controllability type in [8] we have proposed such sort of constructive procedures. It is necessery now to extend them onto the the property (SD) and other abovementioned definitions of harmonization taking into account the basic recommendations of United Nations Organization whichare vitally important for regions too [9].

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